Perturbative Stability of the QCD Predictions for Single Spin Asymmetry in Heavy Quark Photoproduction[*]

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We present the threshold resummation for the single spin asymmetry in heavy flavor production by linearly polarized photons. We analyze the soft-gluon contributions to the asymmetry at fixed order in α_s to the next-to-leading logarithmic accuracy. Our analysis shows that, contrary to the production cross section, the azimuthal asymmetry is practically insensitive to soft radiation. Our calculations of the asymmetry up to the 6th order in α_s lead only to small corrections (of order of few percent) to the Born predictions at energies of the fixed target experiments. Fast convergence of the perturbative series for the azimuthal asymmetry is due to the factorization properties of the photon-hadron cross section. We conclude that measurements of the single spin asymmetry would provide an excellent test of pQCD applicability to heavy flavor production.

I. INTRODUCTION

Presently, the basic spin-averaged characteristics of heavy flavor hadro-, photo- and electroproduction are known exactly up to the next-to-leading order (NLO) (see [1] for a review). Two main results of the explicit pQCD calculations can be formulated as follows. First, the NLO corrections are large; they increase the leading order (LO) predictions for both charm and bottom production cross sections approximately by a factor of 2. For this reason, one could expect that the higher order corrections as well as the nonperturbative contributions can be essential in these processes, especially for the c-quark case. Second, the one-loop predictions are very sensitive to standard uncertainties in the input QCD parameters. In fact, the total uncertainties associated with the unknown values of the heavy quark mass, m, the factorization and renormalization scales, μ_F and μ_R, Λ_{QCD} and the parton distribution functions are so large that one can only estimate the order of magnitude of the NLO predictions for total cross sections [2,3]. For this reason, it is very difficult to compare directly, without additional assumptions, the NLO predictions for spin-averaged cross sections with experimental data and thereby to test the pQCD applicability to heavy quark production.

During the recent years, the role of higher order corrections has been extensively investigated in the framework of the soft gluon resummation formalism. For a review see Ref. [4]. Soft gluon (or threshold) resummation is based on the factorization properties of the cross section near the partonic threshold and makes it possible to resum to all orders in α_s the leading (Sudakov double) logarithms (LL) and the next-to-leading ones (NLL) [5–7]. Formally resummed cross sections are ill-defined due to the Landau pole contribution, and some prescription must be implemented to avoid the renormalon ambiguities. To obtain numerical predictions for the physical cross sections, a few prescriptions have been proposed [8–10]. Physically, the choice of a resummation prescription implies the introduction of an effective scale, separating the hard (perturbative) and soft (nonperturbative) gluon contributions. Another open question, also closely related to the convergence of the perturbative series, is the role of the subleading logarithms which are not, in principle, under control of the resummation procedure [11,12]. Numerically, the higher order corrections to the heavy quark production cross sections can depend significantly on the choice of resummation prescription [12].

Since the spin-averaged characteristics of heavy flavor production are not well defined quantitatively in pQCD it is of special interest to study those spin-dependent observables which are stable under variations of input parameters

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of the theory [13]. In this paper we analyze the charm and bottom production by linearly polarized photons, namely the reactions

$$\gamma \uparrow + h \rightarrow Q + X[\overline{Q}]. \tag{1.1}$$

We consider the single spin asymmetry parameter, $A(S)$, which measures the parallel-perpendicular asymmetry in the quark azimuthal distribution:

$$\frac{\mathrm{d}\sigma_{\gamma h}}{\mathrm{d}\varphi}(S, \varphi) = \frac{\sigma_{\gamma h}^{\mathrm{unp}}(S)}{2\pi}(1 + A(S)\mathcal{P}_\gamma \cos 2\varphi), \tag{1.2}$$

where

$$A(S) = \frac{1}{\mathcal{P}_\gamma} \frac{\mathrm{d}\sigma_{\gamma h}(S, \varphi = 0) - \mathrm{d}\sigma_{\gamma h}(S, \varphi = \pi/2)}{\mathrm{d}\sigma_{\gamma h}(S, \varphi = 0) + \mathrm{d}\sigma_{\gamma h}(S, \varphi = \pi/2)}. \tag{1.3}$$

Here $\sigma_{\gamma h}^{\mathrm{unp}}(S)$ is the unpolarized cross section, $\mathrm{d}\sigma(S, \varphi) \equiv \frac{\mathrm{d}\sigma}{\mathrm{d}\varphi}(S, \varphi)$, $\mathcal{P}_\gamma$ is the degree of linear polarization of the incident photon beam, $\sqrt{S}$ is the centre of mass energy of the process (1.1) and φ is the angle between the beam polarization direction and the observed quark transverse momentum.

The Born level predictions for the single spin asymmetry in (1.1) as well as the contributions of nonperturbative effects (such as the gluon transverse motion in the target and the heavy quark fragmentation) have been considered in [13]. The following remarkable properties of $A(S)$ have been observed:

- The azimuthal asymmetry (1.3) is of leading twist; in a wide region of initial energy, it is predicted to be about 20% for both charm and bottom quark production.

- At energies sufficiently above the production threshold, the LO predictions for $A(S)$ are insensitive (to within few percent) to uncertainties in the QCD input parameters: m, μ_R, μ_F, Λ_{QCD} and in the gluon distribution function. This implies that theoretical uncertainties in the spin-dependent and spin-averaged cross sections (the numerator and denominator of the fraction (1.3), respectively) cancel each other with a good accuracy.

- Nonperturbative corrections to the b-quark azimuthal asymmetry $A(S)$ due to the gluon transverse motion in the target are negligible. Because of the smallness of the c-quark mass, the analogous corrections to $A(s)$ in the charm case are larger; they are of the order of 20%.

In the present paper, the soft-gluon corrections to the asymmetry (1.3) are investigated. Using the methods of Refs. [5–7] for the resummation of soft gluons, we resum the Sudakov logarithms for the polarized cross section (1.2) in moment space to the NLL accuracy. Considering the obtained expression as a generating functional of the perturbative theory, we re-expand it in α_s in momentum space and analyze the fixed order predictions for the azimuthal asymmetry. Our main results can be formulated as follows:

- Contrary to the production cross sections, the asymmerty (1.3) in azimuthal distributions of both charm and bottom quark is practically insensitive to radiative corrections at fixed target energies. This implies that large soft-gluon contributions to the spin-dependent and unpolarized cross sections cancel each other in (1.3) with a good accuracy.

- At the NLL level, the NLO and NNLO predictions for $A(S)$ affect the LO results by less than 1% and 2%, respectively.

- Our computations of higher order corrections indicate a fast convergence of the perturbative series for $A(S)$. We have calculated the asymmetry parameter $A(S)$ up to the 6th order (N^{6}LO) in α_s to the NNL accuracy and have found that corresponding corrections to the Born level result are of order of few percent.

Our analysis shows that high perturbative stability of $A(S)$ is due to the factorization properties of the photon-hadron cross section. We conclude that, in contrast with the production cross sections, the single spin asymmetry in heavy flavor photoproduction is an observable quantitatively well defined in pQCD: it is stable, both parametricaly

and perturbatively, and insensitive to nonperturbative corrections. Measurements of the azimuthal asymmetry in bottom photoproduction would provide an ideal test of the conventional parton model based on pQCD.

Concerning the experimental aspects, the azimuthal asymmetry in charm photoproduction can be measured at SLAC where a coherent bremsstrahlung beam of linearly polarized photons with energies up to 40 GeV will be available soon [14,15]. Due to the c-quark low mass, data on the D-meson azimuthal distributions would make it possible to clarify the role of subleading twist contributions [13].

The paper is organized as follows. Section II contains the derivation of the resummed formula for the spin-dependent photoproduction of heavy flavor in the single-particle inclusive kinematics. In Section III we analyze the NLO and NNLO predictions for the azimuthal asymmetry. We check the quality of the NNL approximation against available explicit results and discuss the subleading logarithms contribution. Higher order corrections and the role of the gluon distribution function in perturbative stability of the asymmetry are considered in Section IV.

II. SOFT GLUON RESUMMATION FOR POLARIZED CROSS SECTION

In this Section we carry out the resummation of the Sudakov logarithms for the spin-dependent cross section of the reaction

$$\gamma(k_\gamma) \uparrow + h(k_h) \rightarrow Q(p_Q) + X[\overline{Q}](p_X) \tag{2.1}$$

to next-to-leading logarithmic (NLL) accuracy to all orders of the perturbative expansion. In the single-particle inclusive (1PI) kinematics, the overall invariants are defined as

$$
\begin{aligned}
S &= (k_\gamma + k_h)^2; & T_1 &= (k_h - p_Q)^2 - m^2; \\
S_4 &= S + T_1 + U_1; & U_1 &= (k_\gamma - p_Q)^2 - m^2.
\end{aligned}
\tag{2.2}
$$

At the parton level, the dominant subprocess is the photon-gluon fusion:

$$\gamma(k_\gamma) + g(k_g) \rightarrow Q(p_Q) + X[\overline{Q}](p_X), \tag{2.3}$$

where the corresponding kinematical variables are

$$
\begin{aligned}
s &= (k_\gamma + k_g)^2 = zS; & t_1 &= (k_g - p_Q)^2 - m^2 = zT_1; \\
s_4 &= s + t_1 + u_1; & u_1 &= U_1,
\end{aligned}
\tag{2.4}
$$

with $k_g = zk_h$, m the heavy quark mass, while s_4 measures the inelasticity of the partonic reaction. At the Born level, $\mathcal{O}(\alpha_{em}\alpha_s)$, the only partonic subprocess which is responsible for heavy quark photoproduction is the two-body photon-gluon fusion:

$$\gamma(k_\gamma) + g(k_g) \rightarrow Q(p_Q) + \overline{Q}(p_{\overline{Q}}). \tag{2.5}$$

To take into account the NLO contributions, one needs to calculate the virtual $\mathcal{O}(\alpha_{em}\alpha_s^2)$ corrections to the Born process (2.5) and the real gluon emission:

$$\gamma(k_\gamma) + g(k_g) \rightarrow Q(p_Q) + \overline{Q}(p_{\overline{Q}}) + g(p_g). \tag{2.6}$$

We neglect the photon-(anti)quark fusion subprocesses as well as the so-called hadronic or resolved component of the photon. This is justified as their contributions vanish at LO and are small at NLO at the energies under consideration [1,2].

The factorized single-heavy quark inclusive cross section for photon-hadron collisions, $d\sigma_{\gamma h}$, has the form of a convolution of the perturbative short-distance cross section, $d\hat{\sigma}_{\gamma g}$, with the universal parton distribution function $\phi_{g/h}$:

$$\mathrm{d}\sigma_{\gamma h}\left(S, T_1, U_1, \varphi, \mu_F, \alpha_s(\mu_R^2)\right) = \int \mathrm{d}z \phi_{g/h}(z, \mu_F) \mathrm{d}\hat{\sigma}_{\gamma g}\left(s, t_1, u_1, \varphi, \mu_F, \alpha_s(\mu_R^2)\right), \tag{2.7}$$

where μ_F and μ_R are the factorization and renormalization scales, respectively. Note that here and in the following $\mathrm{d}\sigma_{\gamma h}$ and $\mathrm{d}\hat{\sigma}_{\gamma g}$ denote any relevant φ-dependent differential distribution. Replacing the incoming hadron by the gluon and taking the Laplace moments, the above convolution simplifies to a product:

$$\mathrm{d}\tilde{\sigma}_{\gamma g}(N, \varphi) = \tilde{\phi}_{g/g}(N_u) \, \mathrm{d}\hat{\sigma}_{\gamma g}(N, \varphi). \tag{2.8}$$

The moments for $\mathrm{d}\hat{\sigma}_{\gamma g}$ are defined by

$$\mathrm{d}\hat{\sigma}_{\gamma g}(N, \varphi) = \int_0^\infty \frac{\mathrm{d}s_4}{m^2} \mathrm{e}^{-N s_4/m^2} \mathrm{d}\hat{\sigma}_{\gamma g}(s_4, \varphi), \tag{2.9}$$

with N the moment variable. The upper limit of this integral is not important for large N and may be put at 1. Similarly, the moments for $\phi_{g/g}$ with respect to z have the form

$$\tilde{\phi}_{g/g}(N_u) = \int_0^1 \mathrm{d}z \mathrm{e}^{-N_u z} \phi_{g/g}(z), \qquad N_u = N(-u_1/m^2), \tag{2.10}$$

where definition of N_u is given for the 1PI kinematics [6,16].

The short-distance perturbative cross section $\mathrm{d}\hat{\sigma}_{\gamma g}$ still be sensitive to the collinear gluon emission, $\vec{p}_{g,T} \to 0$. To separate these collinear effects from the hard scattering, a refactorization is introduced [17,5]:

$$\mathrm{d}\tilde{\sigma}_{\gamma g}(N, \varphi) = \tilde{\psi}_{g/g}(N_u) H_{\gamma g}(\varphi) \tilde{S}_{\gamma g}\left(\frac{m}{N\mu_F}\right), \tag{2.11}$$

where $\psi_{g/g}$ is the center-of-mass parton distribution that absorb the universal collinear singularities associated with the initial-state gluon while $\tilde{S}_{\gamma g}$ is the soft-gluon function that describes the non-collinear soft gluon emission. The mass of the heavy quarks protects the final state from collinear singularities. The hard-scattering part of cross section, $H_{\gamma g}$, is free of soft-gluon effects and thus independent of N. In the photoproduction case, both $H_{\gamma g}$ and $\tilde{S}_{\gamma g}$ are simply functions, and not matrices in color space, in contrast with heavy quark or jet production in hadron-hadron collisions [7].

Comparing Eqs. (2.8) and (2.11), we derive

$$\mathrm{d}\hat{\sigma}_{\gamma g}(N, \varphi) = \frac{\tilde{\psi}_{g/g}(N_u)}{\tilde{\phi}_{g/g}(N_u)} \tilde{S}_{\gamma g}\left(\frac{m}{N\mu_F}\right) H_{\gamma g}(\varphi). \tag{2.12}$$

The functions $\tilde{\psi}_{g/g}/\tilde{\phi}_{g/g}$ and $\tilde{S}_{\gamma g}$ originate from the collinear, $\vec{p}_{g,T} \to 0$, and soft, $\vec{p}_g \to 0$, limits. Since the azimuthal angle φ is the same for both γg and $Q\overline{Q}$ center-of-mass systems in these limits, only the hard function, $H_{\gamma g}$, is φ- dependent in the right-hand part of (2.12).

In the $\overline{\mathrm{MS}}$ factorization scheme, at NLO and to NLL accuracy, the ratio $\tilde{\psi}_{g/g}/\tilde{\phi}_{g/g}$ is [16]

$$\frac{\tilde{\psi}_{g/g}}{\tilde{\phi}_{g/g}}(N_u, \mu_F) = 1 + \frac{\alpha_s C_A}{\pi}\left[\ln^2 \tilde{N}_u + \left(1 + \ln\frac{\mu_F^2}{m^2}\right)\ln \tilde{N}_u\right], \tag{2.13}$$

where $\tilde{N}_u = N_u \mathrm{e}^{\gamma_E}$ with γ_E the Euler constant. The function $\tilde{\psi}_{g/g}/\tilde{\phi}_{g/g}$ summarizes all leading logarithms of $\tilde{N}$ and a part of the next-to-leading ones. (At NLO, they are $\ln^2 \tilde{N}$ and $\ln \tilde{N}$, respectively). $\tilde{S}_{\gamma g}$-function contributes only at the next-to-leading level of $\ln \tilde{N}$.

To resum the leading, $\mathcal{O}\left(\alpha_s^n \ln^{2n} \tilde{N}\right)$, and next-to-leading, $\mathcal{O}\left(\alpha_s^n \ln^{2n-1} \tilde{N}\right)$, logarithms for $\mathrm{d}\hat{\sigma}_{\gamma g}(N, \varphi)$ to all orders of n, we use the methods of Refs. [5–7] based on the solving of the appropriate evolution equations for each of the functions in (2.12). The formal result for the resummed spin-dependent cross section in moment space is:

$$d\hat{\sigma}_{\gamma g}(N, t_1, u_1, \varphi, \mu_F, \alpha_s(\mu_R^2)) = \exp\left[E_g(N_u)\right] H_{\gamma g}(t_1, u_1, \varphi, \alpha_s(\mu_R^2)) \tilde{S}_{\gamma g}\left(1, \alpha_s\left(m^2/N^2\right)\right) \times$$

$$\exp\left[\int_m^{m/N} \frac{d\mu'}{\mu'} 2\,\mathrm{Re}\Gamma_S^{\gamma g}\left(\alpha_s\left(\mu'^2\right)\right)\right]. \tag{2.14}$$

The first exponent in (2.14) resums the N-dependence of the ratio $\tilde{\psi}_{g/g}/\tilde{\phi}_{g/g}$ and is given in the $\overline{\mathrm{MS}}$ scheme by

$$E_g(N_u) = \int_0^\infty \frac{d\omega}{\omega}\left(1 - e^{-N_u\omega}\right)\left[\int_{\omega^2 m^2}^{\mu_F^2} \frac{d\mu'^2}{\mu'^2} A_g\left(\alpha_s\left(\mu'^2\right)\right) + \frac{1}{2}\nu_g\left(\alpha_s\left(\omega^2 m^2\right)\right)\right]. \tag{2.15}$$

The function A_g is known at two loops [17,18], $A_g(\alpha_s) = \alpha_s C_A/\pi + (\alpha_s/\pi)^2 C_A K/2$, $K = C_A\left(67/18 - \pi^2/6\right) - 5n_f/9$, where n_f is the number of active quark flavors, and $\nu_g = 2\alpha_s C_A/\pi$ [7].

The second exponent controls the evolution of the soft function from scale m/N to m and is given in terms of $\Gamma_S^{\gamma g}$, the soft anomalous dimension. $\Gamma_S^{\gamma g}$ was calculated at one loop in [16]:

$$\Gamma_S^{\gamma g}(\alpha_s) = \frac{\alpha_s C_A}{2\pi}\left[\ln\frac{u_1 t_1}{m^4} + \left(1 - \frac{2C_F}{C_A}\right)(1 + L_\beta)\right]. \tag{2.16}$$

Here

$$L_\beta = \frac{1 - 2m^2/s}{\beta}\left[\ln\left(\frac{1-\beta}{1+\beta}\right) + i\pi\right], \qquad \beta = \sqrt{1 - 4m^2/s}, \tag{2.17}$$

while $C_A = N_c$ and $C_F = (N_c^2 - 1)/(2N_c)$, where N_c is the number of colors. The product $H_{\gamma g}\tilde{S}_{\gamma g}$ on the first line of Eq.(2.14) is determined from matching at lowest order in α_s to the Born result.

The exponent in Eqs.(2.14), (2.15) can only be interpreted in a formal sense since the corresponding integration over ω near Λ_{QCD}/m is not defined unambiguously due to the Landau pole in the coupling strength α_s. To avoid this soft gluon divergence, several prescriptions have been proposed [8–10]. However, if one expands the exponents in the resummed cross section at fixed order in α_s, no divergences associated with the Landau pole are encountered.

III. NLO AND NNLO PREDICTIONS

A. Partonic Cross Sections

In this section we expand the resummed cross section (2.14) to one and two loop order and invert back to momentum space using the equation [19]

$$\left(-\ln\tilde{N}\right)^{l+1} = (l+1)\int_0^\infty ds_4 e^{-Ns_4/m^2}\left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\right]_+ + \mathcal{O}\left(\ln^{l-1}\tilde{N}\right), \tag{3.1}$$

where 1PI singular "plus" distributions are defined by

$$\left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\right]_+ = \lim_{\epsilon\to 0}\left\{\frac{\ln^l\left(s_4/m^2\right)}{s_4}\theta\left(s_4 - \epsilon\right) + \frac{1}{l+1}\ln^{l+1}\left(\frac{\epsilon}{m^2}\right)\delta\left(s_4\right)\right\}. \tag{3.2}$$

For any sufficiently regular test function $h(s_4)$, Eq.(3.2) gives

$$\int_0^{s_4^{\max}} ds_4 h(s_4)\left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\right]_+ = \int_0^{s_4^{\max}} ds_4\left[h(s_4) - h(0)\right]\frac{\ln^l\left(s_4/m^2\right)}{s_4} + \frac{1}{l+1}h(0)\ln^{l+1}\left(s_4^{\max}/m^2\right). \tag{3.3}$$

Eq.(3.1) provides us the NLL approximation for both spin-dependent, $d\Delta\hat{\sigma}_{\gamma g}$, and unpolarized, $d\hat{\sigma}_{\gamma g}$, differential cross sections:

$$s^2 \frac{\mathrm{d}^3 \hat{\sigma}_{\gamma g}\left(s, t_1, u_1, \varphi\right)}{\mathrm{d}\varphi \mathrm{d}t_1 \mathrm{d}u_1} = \frac{1}{2\pi}\left(s^2 \frac{\mathrm{d}^2 \hat{\sigma}_{\gamma g}\left(s, t_1, u_1\right)}{\mathrm{d}t_1 \mathrm{d}u_1} + s^2 \frac{\mathrm{d}^2 \Delta\hat{\sigma}_{\gamma g}\left(s, t_1, u_1\right)}{\mathrm{d}t_1 \mathrm{d}u_1} \mathcal{P}_\gamma \cos 2\varphi\right), \tag{3.4}$$

where $\mathcal{P}_\gamma$ is the degree of the photon beam polarization; φ is the angle between the observed quark transverse momentum, $\vec{p}_{Q,T}$, and the beam polarization direction.

To NLL accuracy, the perturbative expansion for the partonic cross section can be written in a factorized form as

$$s^2 \frac{\mathrm{d}^2(\Delta)\hat{\sigma}_{\gamma g}\left(s, t_1, u_1\right)}{\mathrm{d}t_1 \mathrm{d}u_1} = (\Delta)B_{\gamma g}^{\mathrm{Born}}\left(s, t_1, u_1\right)\left\{\delta\left(s + t_1 + u_1\right) + \sum_{n=1}^{\infty}\left(\frac{\alpha_s C_A}{\pi}\right)^n K^{(n)}\left(s, t_1, u_1\right)\right\}, \tag{3.5}$$

with the Born level hard parts $(\Delta)B_{\gamma g}^{\mathrm{Born}}$ given by [20,13]

$$B_{\gamma g}^{\mathrm{Born}}\left(s, t_1, u_1\right) = 2\pi e_Q^2 \alpha_{em} \alpha_s T_F\left[\frac{t_1}{u_1} + \frac{u_1}{t_1} + \frac{4m^2 s}{t_1 u_1}\left(1 - \frac{m^2 s}{t_1 u_1}\right)\right], \tag{3.6}$$

$$\Delta B_{\gamma g}^{\mathrm{Born}}\left(s, t_1, u_1\right) = 2\pi e_Q^2 \alpha_{em} \alpha_s T_F\left[\frac{4m^2 s}{t_1 u_1}\left(1 - \frac{m^2 s}{t_1 u_1}\right)\right]. \tag{3.7}$$

In (3.6) and (3.7), $T_F = \mathrm{Tr}(T^a T^a)/(N_c^2 - 1) = 1/2$ is the color factor, and e_Q is the quark charge in units of electromagnetic coupling constant.

The NLO and NNLO soft gluon corrections to NLL accuracy are

$$K^{(1)}\left(s, t_1, u_1\right) = 2\left[\frac{\ln\left(s_4/m^2\right)}{s_4}\right]_+ - \left[\frac{1}{s_4}\right]_+ \left\{1 + \ln\left(\frac{u_1}{t_1}\right) - \left(1 - \frac{2C_F}{C_A}\right)\left(1 + \mathrm{Re}L_\beta\right) + \ln\left(\frac{\mu^2}{m^2}\right)\right\} +$$
$$\delta\left(s_4\right)\ln\left(\frac{-u_1}{m^2}\right)\ln\left(\frac{\mu^2}{m^2}\right), \tag{3.8}$$

and

$$K^{(2)}\left(s, t_1, u_1\right) = 2\left[\frac{\ln^3\left(s_4/m^2\right)}{s_4}\right]_+ -$$
$$3\left[\frac{\ln^2\left(s_4/m^2\right)}{s_4}\right]_+ \left\{1 + \ln\left(\frac{u_1}{t_1}\right) - \left(1 - \frac{2C_F}{C_A}\right)\left(1 + \mathrm{Re}L_\beta\right) + \frac{2}{3}\frac{b_2}{C_A} + \ln\left(\frac{\mu^2}{m^2}\right)\right\} +$$
$$2\left[\frac{\ln\left(s_4/m^2\right)}{s_4}\right]_+ \left\{1 + \ln\left(\frac{u_1}{t_1}\right) - \left(1 - \frac{2C_F}{C_A}\right)\left(1 + \mathrm{Re}L_\beta\right) + \ln\left(\frac{-u_1}{m^2}\right) + \frac{b_2}{C_A} + \frac{1}{2}\ln\left(\frac{\mu^2}{m^2}\right)\right\} \times$$
$$\ln\left(\frac{\mu^2}{m^2}\right) - \left[\frac{1}{s_4}\right]_+ \ln^2\left(\frac{\mu^2}{m^2}\right)\left\{\ln\left(\frac{-u_1}{m^2}\right) + \frac{1}{2}\frac{b_2}{C_A}\right\}, \tag{3.9}$$

where we use $\mu = \mu_F = \mu_R$, and b_2 is the first coefficient of the $\beta\left(\alpha_s\right)$-function expansion:

$$\beta\left(\alpha_s\right) \equiv \frac{\mathrm{d}\ln\alpha_s\left(\mu^2\right)}{\mathrm{d}\ln\mu^2} = -\sum_{k=1}^{\infty} b_{k+1}(\alpha_s/\pi)^k, \tag{3.10}$$

$b_2 = \left(11C_A - 2n_f\right)/12$, $b_3 = \left(34C_A^2 - 10C_A n_f - 16C_A n_f\right)/48$. In (3.8) and (3.9), we have preserved the NLL terms for all the orders of the scale-dependent logarithm $\ln^k\left(\mu^2/m^2\right)$, $(k = 0, 1, 2)$. We have checked that the result (3.8) agrees to NLL accuracy with the exact $\mathcal{O}(\alpha_{em}\alpha_s^2)$ calculations of the unpolarized photon-gluon fusion given in [22,23], and that the Eqs.(3.8) and (3.9) coincide completely with the corresponding ones obtained in [16,21] for the electroproduction case.

To perform a numerical investigation of the results (3.8) and (3.9), it is convenient to introduce for the fully inclusive (integrated over t_1 and u_1) cross sections, $\hat{\sigma}_{\gamma g}$ and $\Delta\hat{\sigma}_{\gamma g}$,

$$(\Delta)\hat{\sigma}_{\gamma g}(s) = \int_{(1-\beta)s/2}^{(1+\beta)s/2} \mathrm{d}(-t_1) \int_0^{s_4^{\max}} \mathrm{d}s_4 \frac{\mathrm{d}^2(\Delta)\hat{\sigma}_{\gamma g}}{\mathrm{d}t_1 \mathrm{d}s_4}(s, t_1, s_4), \tag{3.11}$$

$$s_4^{\max} = s + t_1 + \frac{m^2 s}{t_1}, \tag{3.12}$$

the dimensionless coefficient functions $(\Delta)c^{(k,l)}$,

$$(\Delta)\hat{\sigma}_{\gamma g}(\eta, \mu^2) = \frac{\alpha_{em}\alpha_s(\mu^2)e_Q^2}{m^2} \sum_{k=0}^{\infty} \left(4\pi\alpha_s(\mu^2)\right)^k \sum_{l=0}^{k} (\Delta)c^{(k,l)}(\eta) \ln^l \frac{\mu^2}{m^2}, \qquad \eta = \frac{s}{4m^2} - 1, \tag{3.13}$$

where the variable η measures the distance to the partonic threshold.

Concerning the scale-independent coefficient functions, only $c^{(1,0)}$ is known exactly [24]. As to the μ-dependent coefficients, some of them can by calculated explicitly using the renormalization group equation:

$$\frac{\mathrm{d}(\Delta)\hat{\sigma}_{\gamma g}^{\mathrm{res}}(s,\mu)}{\mathrm{d}\ln\mu^2} = -\int_\rho^1 \mathrm{d}z (\Delta)\hat{\sigma}_{\gamma g}^{\mathrm{res}}(zs,\mu)P_{gg}(z), \tag{3.14}$$

where $\rho = 4m^2/s$, $(\Delta)\hat{\sigma}_{\gamma g}^{\mathrm{res}}(s,\mu)$ are the cross sections resummed to all orders in α_s and $P_{gg}(z)$ is the corresponding (resummed) Altarelli-Parisi gluon-gluon splitting function. Expanding Eq.(3.14) in α_s, we find

$$(\Delta)c^{(1,1)} = \frac{1}{4\pi^2} \left[b_2 (\Delta)c^{(0,0)} - (\Delta)c^{(0,0)} \otimes P_{gg}^{(0)} \right], \tag{3.15}$$

$$(\Delta)c^{(2,1)} = \frac{1}{(4\pi^2)^2} \left[b_3 (\Delta)c^{(0,0)} - (\Delta)c^{(0,0)} \otimes P_{gg}^{(1)} \right] + \frac{1}{4\pi^2} \left[2b_2 (\Delta)c^{(1,0)} - (\Delta)c^{(1,0)} \otimes P_{gg}^{(0)} \right], \tag{3.16}$$

$$(\Delta)c^{(2,2)} = \frac{1}{4\pi^2} \left[b_2 (\Delta)c^{(1,1)} - \frac{1}{2}(\Delta)c^{(1,1)} \otimes P_{gg}^{(0)} \right] = \tag{3.17}$$
$$\frac{1}{(4\pi^2)^2} \left[b_2 (\Delta)c^{(0,0)} - \frac{3}{2}b_2 (\Delta)c^{(0,0)} \otimes P_{gg}^{(0)} + \frac{1}{2}(\Delta)c^{(0,0)} \otimes P_{gg}^{(0)} \otimes P_{gg}^{(0)} \right],$$

where the convolutions $(\Delta)c^{(k,l)} \otimes P_{gg}^{(j)}$ are defined as

$$\left[(\Delta)c^{(k,l)} \otimes P_{gg}^{(j)} \right](s) \equiv \int_\rho^1 \mathrm{d}z (\Delta)c^{(k,l)}(zs)P_{gg}^{(j)}(z), \tag{3.18}$$

with the one- and two-loop gluon splitting functions, $P_{gg}(z) = (\alpha_s/\pi)P_{gg}^{(0)}(z) + (\alpha_s/\pi)^2 P_{gg}^{(1)}(z) + ...$, given in [25]. Note that Eq.(3.15) agrees with the exact result for $c^{(1,1)}$ given in [24].

With Eqs.(3.15)-(3.17) in hand, we are able to check the quality of the NLL approximation at both NLO and NNLO against exact answers. In Figs.1 and 2 we plot the functions $c^{(k,l)}(\eta)$ and $\Delta c^{(k,l)}(\eta)$, respectively. Predictions of the NLL approximation (3.8), (3.9) are given by dashed curves. The available exact results are given by solid lines. One can see a reasonable agreement up to energies $\eta \approx 2$.

In Fig.3 we plot the NLL predictions for the ratios $\frac{\Delta c^{(n,0)}}{c^{(n,0)}}(\eta)$, $n = 0, 1, 2$. The NNLO curve is given only up to $\eta = 2$ since, at larger energies, the coefficient function $c^{(2,0)}$ changes sign and the ratio $\frac{\Delta c^{(2,0)}}{c^{(2,0)}}$ strongly oscillates. Fig.3 shows sizable deviations of the NLO and NNLO results from the Born level ones. This is due to the fact that the physical soft-gluon corrections (3.11) are determined by a convolution of the Born cross section with the Sudakov logarithms which, apart from factorized $\delta(s_4)$-terms, contain also non-factorable ones (see Eq.(3.3)). Kinematically, large values of η allow $s_4/m^2 \gtrsim 1$ that leads to significant non-factorable corrections. In other words, the collinear bremsstrahlung carries away a large part of the initial energy. Since the spin-dependent and unpolarized Born level partonic cross sections have different energy behavior, the soft radiation has different impact on these quantities.
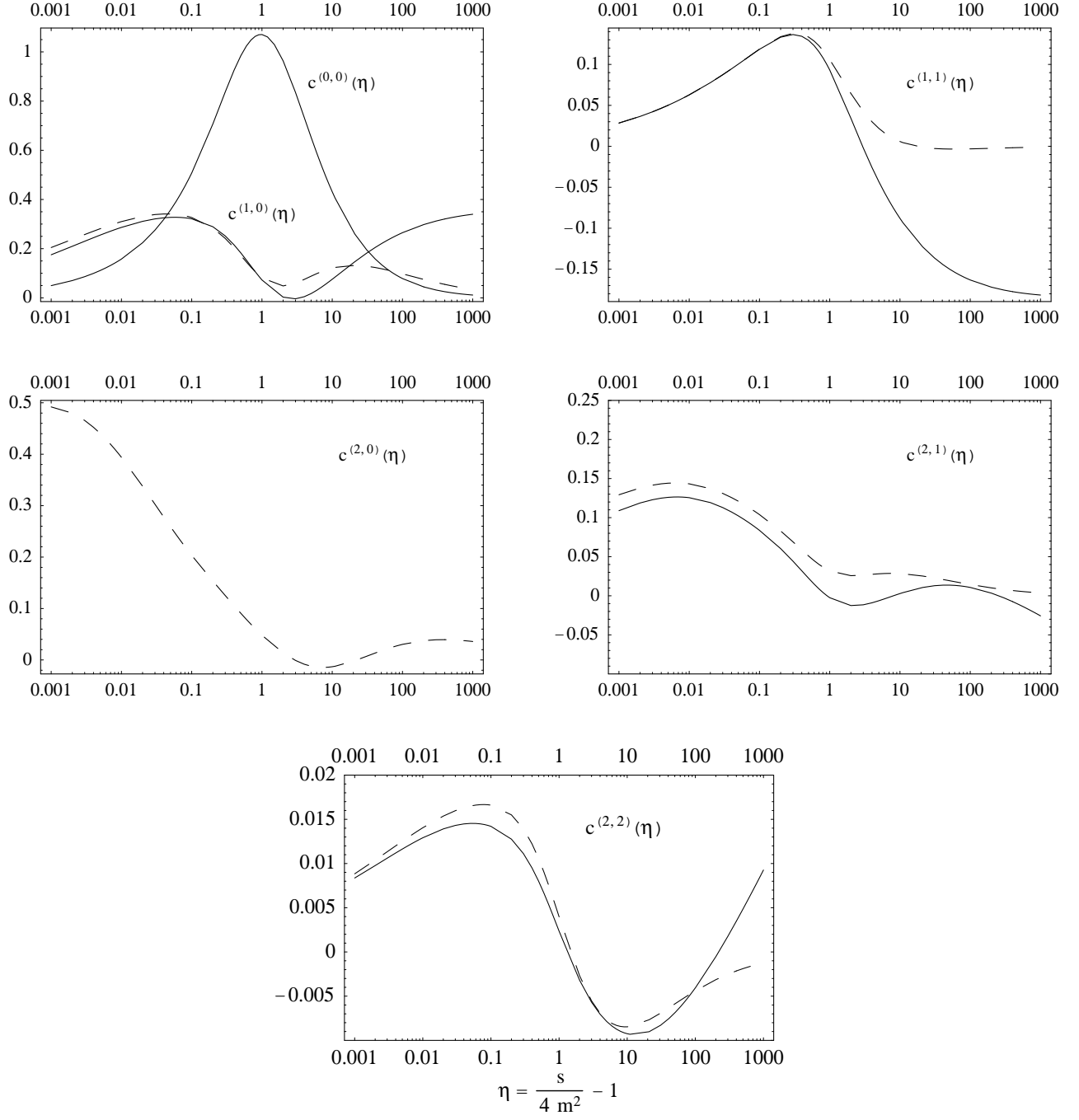
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FIG. 1. $c^{(k,l)}(\eta)$ coefficient functions. Plotted are the available exact results (solid lines) and the NLL approximation (dashed lines).
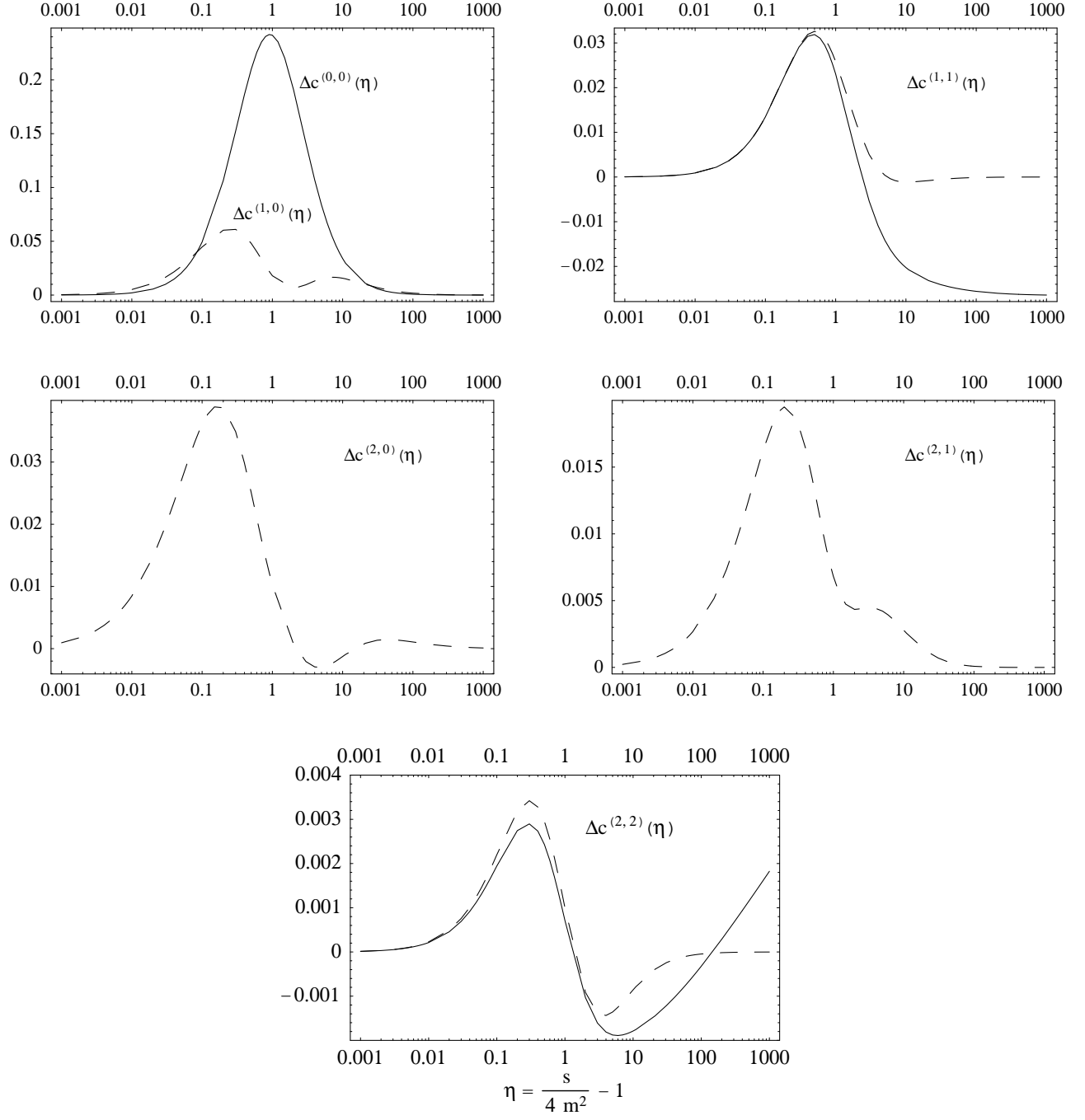
8

FIG. 2. $\Delta c^{(k,l)}(\eta)$ coefficient functions. Plotted are the available exact results (solid lines) and the NLL approximation (dashed lines).
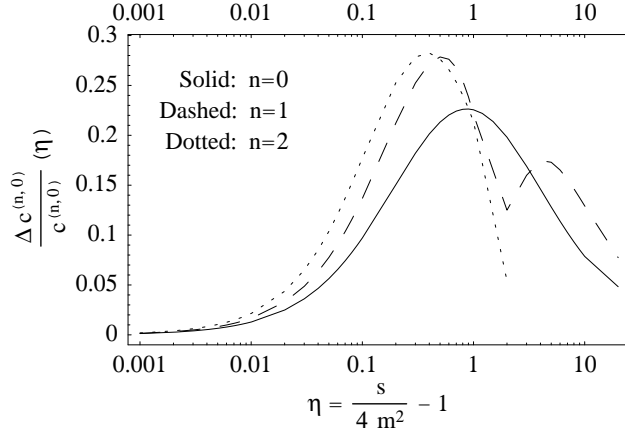
FIG. 3. NLL predictions for the ratio $\frac{\Delta c^{(n,0)}}{c^{(n,0)}}(\eta)$. Plotted are the LO (solid curve), NLO (dashed curve) and NNLO (dotted curve) results.

B. Hadron Level Results

Let us now analyze the impact of the approximate NLO and NNLO perturbative corrections on the azimuthal asymmetry, $A(S)$, at hadron level. Unless otherwise stated, the CTEQ5M [26] parametrization of the gluon distribution function is used. The default values of the charm and bottom mass are $m_c = 1.5$ GeV and $m_b = 4.75$ GeV. For our analysis at NLO (NNLO) we use the two-loop (three-loop [28]) expression for α_s; $\Lambda_4 = 300$ MeV and $\Lambda_5 = 200$ MeV. The default values of the factorization scale μ_F chosen for the $A(S)$ asymmetry calculation are $\mu_F \mid_{\text{Charm}} = 2m_c$ for the case of charm production and $\mu_F \mid_{\text{Bottom}} = m_b$ for the bottom case [1,29]. For the renormalization scale, μ_R, we use $\mu_R = \mu_F$.

Our results for the single spin asymmetry $A(S)$ in charm and bottom photoproduction at fixed target energies are presented in Fig.4. For comparison, we plot in Fig.5 the so-called K-factors for unpolarized cross sections: $K_{\gamma h}^{(1)}(S) = \sigma_{\gamma h}^{\text{NLO}}(S) / \sigma_{\gamma h}^{\text{LO}}(S)$ and $K_{\gamma h}^{(2)}(S) = \sigma_{\gamma h}^{\text{NNLO}}(S) / \sigma_{\gamma h}^{\text{NLO}}(S)$. One can see from Figs.4 and 5 that large soft-gluon corrections to the production cross sections practically (to within 1-2 percent) do not affect the Born predictions for $A(S)$ at both NLO and NNLO.
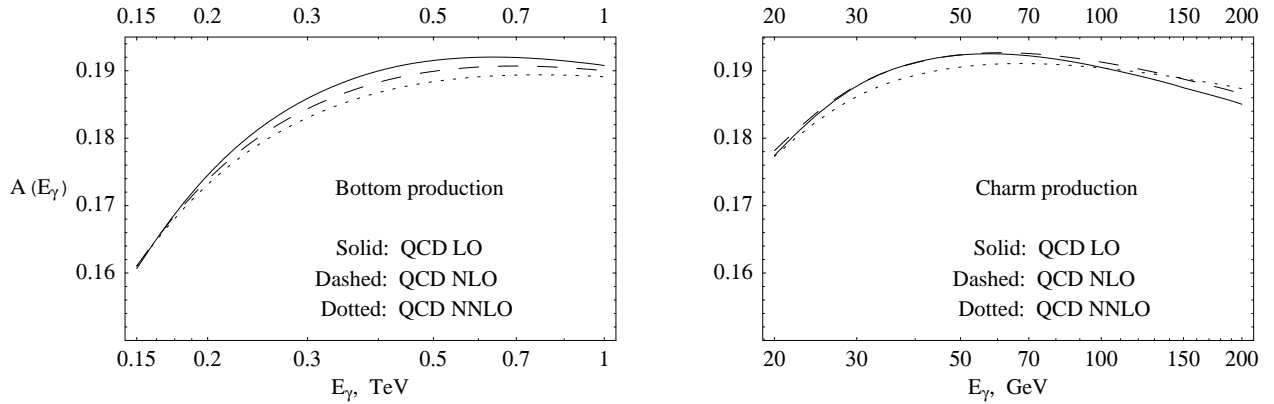

FIG. 4. Single spin asymmetry, $A(E_\gamma)$, in b- and c-quark production at LO (solid curve), NLO (dashed curve) and NNLO (dotted curve) to NLL accuracy.
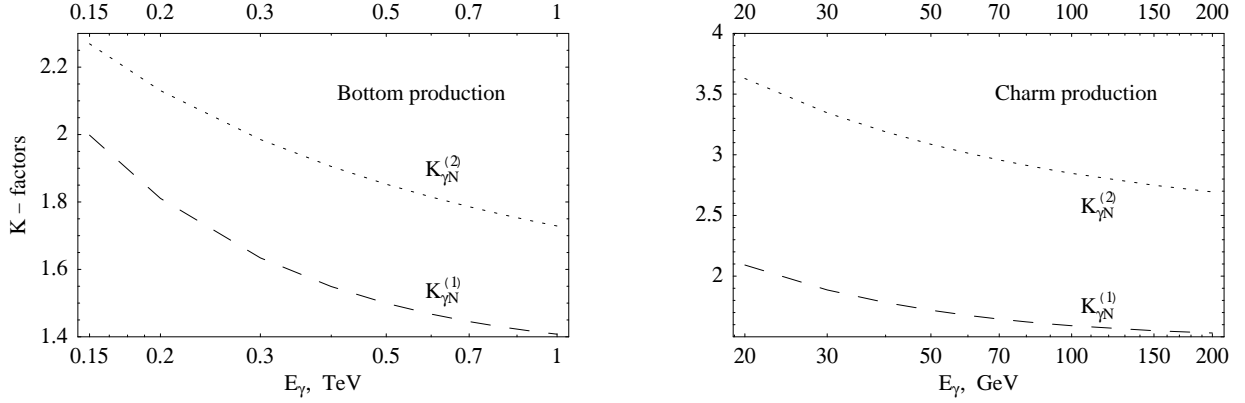
10

FIG. 5. Energy behavior of the $K(E_\gamma)$-factors, $K_{\gamma h}^{(1)}(S) = \sigma_{\gamma h}^{\text{NLO}}(S) / \sigma_{\gamma h}^{\text{LO}}(S)$ (dashed curve) and $K_{\gamma h}^{(2)}(S) = \sigma_{\gamma h}^{\text{NNLO}}(S) / \sigma_{\gamma h}^{\text{NLO}}(S)$ (dotted curve), in b- and c-quark production at NLL level.

Our calculations given in Figs.3-5 represent the central result of this paper. At first sight, the situation seems paradoxial: large soft-gluon corrections display a strong φ-dependence in the case of the photon-gluon fusion and, simultaneously, are practically φ-independent at the hadron level. A qualitative explanation of this fact will be given in Section IV. We shall see that sufficiently soft gluon distribution function leads to a factorization of the photon-hadron cross section and, as a consequence, to fast convergence of the perturbative series for $A(S)$.

Another remarkable property of the azimuthal asymmetry closely related to fast perturbative convergence is its parametric stability[1]. As it was shown in [13], the LO predictions for $A(S)$ are insensitive (to within few percent) to standard theoretical uncertainties in the QCD input parameters: m, μ_R, μ_F, Λ_{QCD} and in the gluon distribution function. We have verified that the same situation takes place at higher orders too. In particular, all the CTEQ5 versions of the gluon density as well as the MRST parametrizations [27] lead to asymmetry predictions which coincide with each other with accuracy better than 1.5%.

Note also the scaling behavior of the azimuthal asymmetry: with a good accuracy the quantity $A(S)$ is a function of the only variable ρ_h, $\rho_h = 4m^2/S$, so that [13]

$$A(S) \mid_{\text{Bottom}} \approx A\left(Sm_c^2/m_b^2\right) \mid_{\text{Charm}} . \tag{3.19}$$

This property of the asymmetry reflects its independence from Λ_{QCD}.

Let us now discuss the hard $(\vec{p}_{g,T} \neq 0)$ and virtual NLO contributions, i.e. radiative corrections to the hard functions, $H_{\gamma g}(s)$ and $\Delta H_{\gamma g}(s)$. Since these contributions do not contain the Sudakov logarithms, they are expected to be small near the threshold. One can see from Fig.1 that it is really the case. Soft radiation describes very well the exact NLO results on the unpolarized photon-gluon fusion at partonic energies up to $\eta \approx 2$. Since the gluon distribution function supports just the threshold region, the soft-gluon contribution dominates the photon-hadron cross section approximately up to $S/4m^2 \sim 10$. Using the exact expression for the γg cross section [24], we have verified that the contribution originating from the region $\eta > 2$ makes only 1-2% from the NLO predictions for the unpolarized bottom production in γp collisions at $E_\gamma \leq 1$ TeV. In other words, radiative corrections to the unpolarized hard function affect the NLL predictions for the asymmetry $A(S)$ by about $+2\%$ at $S/4m^2 \sim 10$ and by less than 1% at lower energies.

Presently, the exact NLO calculations of the φ-dependent cross section of heavy flavor production are not completed [31]. However we can be sure that, at energies not so far from the production threshold, the soft radiation

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[1]Of course, parametric stability of the fixed order results does not imply a fast convergence of the corresponding series. However, a fast convergent series must be parametricaly stable. In particular, it must be μ_R- and μ_F-independent.

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is the dominant perturbative mechanism in the polarized case too. First, the LO predictions for the φ-dependent cross section are large and the Sudakov logarithms have universal, spin-independent structure. For this reason, the polarized heavy quark production has also a strong threshold enhancement. Second, our analysis of the exact scale-dependent polarized cross sections given in Fig.2 confirms with a good accuracy the dominance of the soft-gluon contribution. Third, we have verified that soft radiation is a good approximation to the threshold behavior of the explicit NLO results [32] on heavy quark production in γg collisions with longitudinally polarized initial states too. These facts argue that radiative corrections to the φ-dependent hard function can not also affect significantly the soft-gluon predictions for the asymmetry at the energies under consideration.

At very high energies, $S/4m^2 \gtrsim 10^2$, the dominant NLO production mechanism is the so-called flavor excitation (FE). This mechanism arises from the diagrams with the t-channel gluon exchange and leads to a constant value for the unpolarized photon-gluon fusion cross section as $s \to \infty$ [22,24]. In this limit, the hard component associated with the FE mechanism is large and can, in principle, affect the LO predictions for the asymmetry. One can expect that hard corrections will result in some dilution of the azimuthal asymmetry at superhigh energies because of different asymptotical behavior of the spin-dependent and unpolarized cross sections: at $s \to \infty$, $c_{\mathrm{FE}}^{(1,0)}(s) \propto const$ while $\Delta c_{\mathrm{FE}}^{(1,0)}(s) \propto \ln s/s$. In particular, hard corrections can be sizable in the case of charm production at $E_\gamma \sim 1$ TeV. For this reason, we consider only the energy region up to $S/4m^2 \sim 10$.

In the conclusion of this section, a remark about the next-to-next-to-leading logarithms (NNLL) contribution. In the unpolarized case, it is possible to estimate the contribution of NNLL at NNLO using the method proposed in Refs. [30,12]. For this purpose, we can add to the NLL approximation (3.8) the subleading $\delta(s_4)$-term containing the exact virtual plus soft corrections [22,23]. Exponentiating the obtained expression, one can determine the coefficients of all the powers of the threshold logarithms $[\ln^k (s_4/m^2)/s_4]_+$ at NNLO. (Remember that the NLL expansion (3.9) does not give all the terms with $k = 0, 1$ but only those ones involving the scale μ). Since the exact expression for $\Delta c^{(1,0)}$ is not presently available, in the polarized case we can analyze numerically only the factorized contribution of the so-called Coulomb singularity which originates from the final state interaction between the massive quarks and dominates in the region very close to the threshold [22]. We have verified that the Coulomb corrections to both $\Delta c^{(2,0)}$ and $c^{(2,0)}$ are of the order of few percent and that their contribution to $A(S)$ is negligible.

Beyond the NNLO, not all of the subleading logarithms coefficients are under control in the resummation formalism. At the same time, the NLL predictions for partonic cross sections begin to grow rapidly with the order of the perturbative expansion in α_s due to the renormalon ambiguities [9,10]. For this reason, the perturbative expansion is usually stopped at NNLO, avoiding the theoretical problem with power corrections. Nevertheless, we shall see in the next section that in the case of single spin asymmetry it makes sense to analyze the higher order corrections too.

IV. HIGHER ORDER CONTRIBUTIONS

Formally, the finite-order expansion procedure can be extended at NLL level to arbitrary higher order in α_s. According to (2.14) and (2.15), the LL contribution can be written as

$$\frac{\mathrm{d}\hat{\sigma}_{\gamma g}^{\mathrm{LL}}}{\mathrm{d}\varphi}(s, \varphi) = \int_0^{y_m} \mathrm{d}y\, x_y^{3/2} \frac{\mathrm{d}\hat{\sigma}_{\gamma g}^{\mathrm{Born}}}{\mathrm{d}\varphi}(x_y s, \varphi) \left\{ \delta(y) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s C_A}{\pi} \right)^n \frac{2n}{n!} \left[\frac{\ln^{2n-1} y}{y} \right]_+ \right\}, \tag{4.1}$$

where $y = s_4/m^2$, $x_y = (1 - ym^2/s)^2$, $y_m = (s - 2m\sqrt{s})/m^2$ and

$$\frac{\mathrm{d}\hat{\sigma}_{\gamma g}^{\mathrm{Born}}}{\mathrm{d}\varphi}(s, \varphi) = \frac{e_Q^2 \alpha_{em} \alpha_s}{2s} \left\{ -2\beta + 2\ln \frac{1+\beta}{1-\beta} + \left[-2\beta(1-\beta^2) + (1-\beta^4)\ln \frac{1+\beta}{1-\beta} \right] (1 + \mathcal{P}_\gamma \cos 2\varphi) \right\} \tag{4.2}$$

with $\beta = \sqrt{1 - 4m^2/s}$. At hadronic level, we have

$$\Delta\sigma_{\gamma h}^{\mathrm{LL}}(S) = \Delta\sigma_{\gamma h}^{\mathrm{Born}}(S) + \sum_{n=1}^{\infty} \left(\frac{\alpha_s C_A}{\pi} \right)^n \frac{2n}{n!} A_{2n-1}(S) \sigma_{\gamma h}^{(2n-1)}(S), \tag{4.3}$$

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where

$$(\Delta)\sigma_{\gamma h}^{(i)}(S) = \frac{1}{S}\int_{4m^2}^{S} ds \phi_{g/h}(s/S)\int_0^{y_m} dy\, x_y^{3/2}(\Delta)\hat{\sigma}_{\gamma g}^{\text{Born}}(x_y s)\left[\frac{\ln^i y}{y}\right]_+ , \tag{4.4}$$

with $\phi_{g/h}(z)$ the gluon distribution function, and the quantity

$$A_i(S) = \frac{\Delta\sigma_{\gamma h}^{(i)}(S)}{\sigma_{\gamma h}^{(i)}(S)} \tag{4.5}$$

describes the partial contribution of the ith power of the threshold logarithm.

We have calculated numerically the quantities $A_i(S)$ up to $i = 11$, i.e., up to the 6th order (N⁶LO) of perturbative expansion in α_s. The results of our computations are presented in Fig.6. One can see that $A_i(S)$ are close to the LO result, $A_{\text{Born}}(S) = \Delta\sigma_{\gamma h}^{\text{Born}}(S)/\sigma_{\gamma h}^{\text{Born}}(S)$, for all $i \leq 11$.
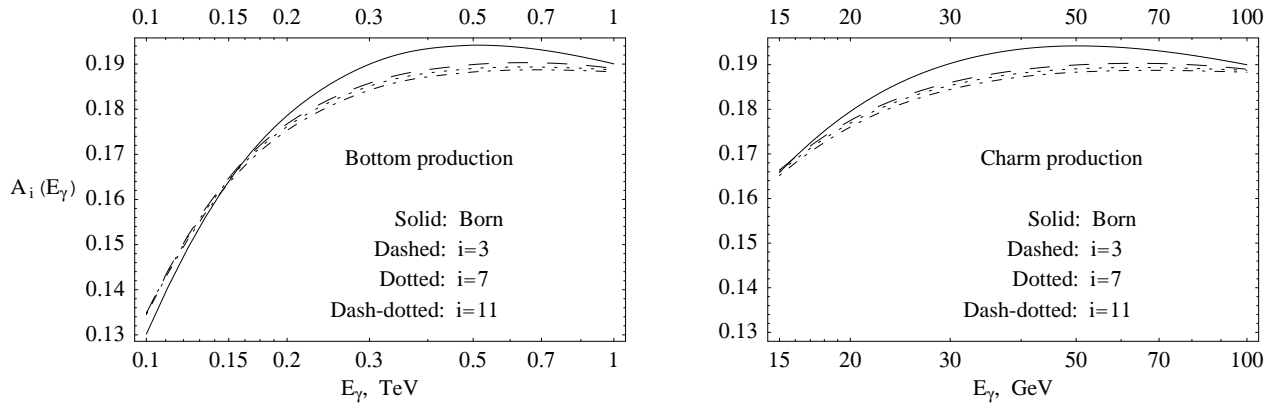

FIG. 6. Energy behavior of the $A_i(E_\gamma)$-factors in b- and c-quark production.

So, we see that, at higher orders of perturbation theory, the LL predictions for the azimuthal asymmetry are also practically insensitive to soft radiation. The same situation takes place at the NLL level too. Indeed, neglecting in (2.14)-(2.16) the small contributions of order of $(1 - 2C_F/C_A) = 1/N_c^2$ and the terms proportional to $\ln(t_1/u_1)$ which vanish after integration over t_1 and u_1, we can write

$$\Delta\sigma_{\gamma h}^{\text{NLL}}(S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s C_A}{\pi}\right)^n \frac{2n-1}{(n-1)!}\left(1 + (n-1)\frac{2b_2}{3C_A}\right)A_{2n-2}(S)\sigma_{\gamma h}^{(2n-2)}(S) + \mathcal{O}\left(1/N_c^2\right), \tag{4.6}$$

with the partial contributions $A_i(S)$ defined by (4.5). Moreover, one can conclude from Eqs.(4.3)-(4.5) and Fig.6 that all those subleading logarithmic (SLL) contributions which have the form

$$(\Delta)\sigma_{\gamma h}^{\text{SLL}}(S) \simeq \sum_{n,i}\left(\frac{\alpha_s}{\pi}\right)^n \frac{C(n,i)}{S}\int_{4m^2}^{S} ds\,\phi_{g/h}(s/S)\int_0^{y_m} dy\, x_y^{3/2}(\Delta)\hat{\sigma}_{\gamma g}^{\text{Born}}(x_y s)\left[\frac{\ln^i y}{y}\right]_+ , \tag{4.7}$$

with arbitrary numerical coefficients $C(n,i)$, also can not affect essentially the LO predictions for the azimuthal asymmetry.

We have also investigated how the soft-gluon corrections to the asymmetry depend on the choice of the gluon distribution function. Parametrizing the gluon density as $z^{-\lambda}(1-z)^L$, we have varied the parameters λ and L in wide intervals: $0 \leq \lambda \leq 1$, $0 \leq L \leq 6$. The energy behavior of the functions $A_i(S)$ was found to be different at extreme values of L and λ however, at $L \geq 2$, the ratio $A_i(S)/A_{\text{Born}}(S)$ is equal to unity with a good accuracy

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irrespective of λ. Moreover, the softer the gluon density (i.e., the larger L) the smaller the corrections to the asymmetry. All curves given in Fig.6 are calculated at $\lambda = 1$, $L = 5$.[2]

To clarify the origin of perturbative stability of the asymmetry, let us rewrite the Eqs.(4.1)-(4.4) in a more convenient way. Changing the order of integrations in (4.4), we obtain

$$(\Delta)\sigma_{\gamma h}^{\mathrm{LL}}(S) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s C_A}{\pi}\right)^n \frac{1}{n!} \int_{4m^2/S}^{1} \mathrm{d}z \phi_{g/h}(z)(\Delta)\hat{\sigma}_{\gamma g}^{\mathrm{Born}}(zS) \, \Phi_n(z, S), \tag{4.8}$$

where

$$\Phi_n(z, S) = -z^{3/2}\phi_{g/h}^{-1}(z)\frac{\partial}{\partial z}\int_z^1 \frac{\mathrm{d}\tau}{\tau^{3/2}}\phi_{g/h}(\tau)\ln^{2n}\left[\left(\tau - \sqrt{z\tau}\right)S/m^2\right] \tag{4.9}$$

$$= \lim_{\epsilon \to 0}\left[\ln^{2n}\frac{\epsilon S}{2m^2} + nz\phi_{g/h}^{-1}(z)\int_{z+\epsilon}^{1}\frac{\mathrm{d}\tau}{\tau}\phi_{g/h}(\tau)\frac{\ln^{2n-1}\left[\left(\tau - \sqrt{z\tau}\right)S/m^2\right]}{\tau - \sqrt{z\tau}}\right]$$

$$= \ln^{2n}\frac{(1-\sqrt{z})S}{m^2} + 2n\int_0^{1-\sqrt{z}}\frac{\mathrm{d}\xi}{\xi}\ln^{2n-1}\left(\frac{\xi S}{m^2}\right)\left[\frac{z\phi_{g/h}\left((\sqrt{z}/2 + \sqrt{z/4+\xi})^2\right)}{2\phi_{g/h}(z)\left(z/4 + \xi + \sqrt{(z/4+\xi)z/4}\right)} - 1\right].$$

Formally, we can write

$$(\Delta)\sigma_{\gamma h}^{\mathrm{LL}}(S) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s C_A}{\pi}\right)^n \frac{1}{n!}\Phi_n\left((\Delta)z_n(S), S\right) \int_{4m^2/S}^{1} \mathrm{d}z \phi_{g/h}(z)(\Delta)\hat{\sigma}_{\gamma g}^{\mathrm{Born}}(zS), \tag{4.10}$$

with the mean value points $4m^2/S \leq z_n(S), \Delta z_n(S) \leq 1$.

At energies up to $S/4m^2 \sim 10$, the functions $\Phi_n(z, S)$, $n \geq 1$, vary slowly at $z \sim 4m^2/S$ and grow at large $z \sim 1$ as $\ln^{2n}\left(1 - \sqrt{z}\right)$. Since the gluon density rapidly vanishes at large z, $\phi_{g/h}(z) \sim (1-z)^L$, both values $z_n(S)$ and $\Delta z_n(S)$ are of the order of $4m^2/S$. In other words, sufficiently soft gluon distribution function makes the collinear gluon radiation effectively soft at hadron level. In this case, the different high energy behavior of the spin-dependent and unpolarized Born cross sections is irrelevant since practically whole contribution to the radiative corrections to these quantities originates from the threshold region. Furthermore, $\hat{\sigma}_{\gamma g}^{\mathrm{Born}}(s)$ and $\Delta\hat{\sigma}_{\gamma g}^{\mathrm{Born}}(s)$ take their maximal values practically at the same energies: one can see from Figs.1 and 2 that both Born level cross sections have peaks at $\eta \simeq 1$. According to the saddle point arguments, these properties of the gluon distribution function and photon-gluon fusion provide with a good accuracy the equality[3]

$$\Phi_n\left(\Delta z_n(S), S\right) \simeq \Phi_n\left(z_n(S), S\right), \tag{4.11}$$

which leads to spin-independent radiative factor in (4.10).

As noted in previous Section and shown in Fig.3, soft-gluon corrections to the partonic cross section depend essentially on the azimuthal angle φ. So, the mere spin-independent structure of the Sudakov logarithms can not explain our results. Our analysis shows that two more factors are responsible for high perturbative stability of the hadron level asymmetry. First, at the energies under consideration, the gluon distribution function supports the contribution of the threshold region. Second, the extremum point of the Born level cross section, $\eta \simeq 1$, is practically φ-independent.

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[2]The gluon distribution function is presently unknown beyond the NLO. We have verified that, in the case of CTEQ5M parametrization, $A_i(S)/A_{\mathrm{Born}}(S)$ is equal to unity to within 4% for all $i \leq 11$.

[3]Note that Eq.(4.11) does not hold at very high energies since the functions $\Phi_n(z, S)$ vary strongly in whole interval $4m^2/S < z < 1$ at $S/4m^2 \gtrsim 10^2$. In this case, the different high energy behavior of the spin-dependent and unpolarized Born cross sections becomes important and leads to φ-dependent soft-gluon corrections.

V. CONCLUSION

In this paper we have investigated the impact of the soft gluon radiation on the single spin asymmetry in heavy flavor production by linearly polarized photons. Our calculations of the spin-dependent cross section at the NLL level up to the 6th order in α_s show that the azimuthal asymmetry is practically insensitive to the soft-gluon corrections at fixed target energies. Taking into account the remarkable properties of $A(S)$ observed in our previous paper [13], we conclude that, unlike the unpolarized cross sections, the single spin asymmetry in heavy flavor photoproduction is an observable quantitatively well defined in pQCD: it is stable both parametricaly and perturbatively, and insensitive to nonperturbative contributions. This asymmetry is of leading twist and can be measured at SLAC where the linearly polarized photon beam will be available soon [14,15]. Measurements of the azimuthal asymmetry would provide an ideal test of the pQCD applicability to heavy flavor production.

ACKNOWLEDGMENTS

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